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H. S. Tzou and R. V. Howard

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Department of Mechanical Engineering University of Kentucky Lexington, KY 40506-0046	

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EDITED BY
CLARK J. RADCLIFFE
MICHIGAN STATE UNIVERSITY

KON-WELL WANG
PENNSYLVANIA STATE UNIVERSITY

H. S. TZOU
UNIVERSITY OF KENTUCKY

ERIC W. HENDRICKS
NAVAL OCEAN SYSTEMS CENTER

A PIEZOTHERMOELASTIC SHELL THEORY
APPLIED TO ACTIVE STRUCTURES

H. S. Tzou and R. V. Howard
Department of Mechanical Engineering
University of Kentucky
Lexington, Kentucky

ABSTRACT

"Smart" structures with integrated sensors, actuators, and control electronics are of importance to next-generation high-performance structural systems. Piezoelectric materials possess unique electromechanical properties, the direct and converse effects, which can be used in sensor and actuator applications. In this study, piezothermoelastic characteristics of piezoelectric shell continua are studied and applications of the theory to active structures in sensing and control are discussed. A generic piezothermoelastic shell theory for thin piezoelectric shells is derived using the linear piezoelectric theory and Kirchhoff–Love assumptions. It shows that the dynamic equations, in three principal directions, include thermal induced loads as well as conventional electric and mechanical loads. The electric membrane forces and moments induced by the converse effect can be used to control the thermal and mechanical loads. A simplification procedure, based on Lame's parameters and radii of curvatures, is proposed and applications of the theory to 1) a piezoelectric cylindrical shell and 2) a piezoelectric beam are demonstrated.

INTRODUCTION

Development of "smart" structures with integrated sensors, actuators, and control electronics are crucial to next–generation structural systems. New sensor/actuator materials are investigated and new technologies are developed in recent years. Among those commonly used sensor/actuator materials (e.g., piezoelectric materials, shape–memory alloys, electrorheological fluids, electrostrictive materials, magnetostrictive materials, etc.), piezoelectric materials possess unique electromechanical properties (the direct and converse piezoelectric effects) which can be respectively used in sensor and actuator applications (Tzou & Fukuda, 1991; Tzou & Anderson, 1992).

General theories derived from a generic shell continuum can be applied to a broad class shell and non–shell structures (Soedel, 1981). Chau (1986) proposed a variational formulation to describe the electromechanical equilibrium of completely anisotropic piezoelectric shells. Rogacheva (1982,1984a,1984b,1986) studied state equations and boundary conditions of piezoelectric shells polarized along coordinate directions. Senik and Kudriavtsev (1980) formulated the equations of motion for piezoelectric shells transversely polarized. Dökmeci (1978) derived a theory for coated thermopiezoelectric laminae. Tzou and Gadre (1989) proposed a generic theory for multi–layered piezoelectric shell actuators based on equivalent induced strains. Tzou (1991) derived a general distributed sensing and control theory for a generic shell continuum using piezoelectric thin layers. A thin piezoelectric solid finite element with three internal degrees of freedom was formulated and applied to distributed sensing and control of continua (Tzou & Tseng, 1991). Tzou and Zhong (1990) derived a piezoelastic vibration theory for a hexagonal symmetrical piezoelectric thick shell with three effective principal axes; and this theory was applied to distributed shell convolving sensors (Tzou & Zhong, 1991a) and active structural control (Tzou & Zhong, 1991b). In this study, the piezoelastic shell vibration theory is extended to include thermal induced effects due to temperature variations. Piezothermoelastic behaviors of piezoelectric shell continua are investigated.

Based on the linear piezoelectric theory and Kirchhoff–Love assumptions, a generic piezothermoelastic shell theory for thin piezoelectric shells is derived first. A simplification procedure, based on Lame's parameters and radii of curvatures, is proposed and applications of the theory to a number of piezoelectric continua (a piezoelectric cylindrical shell and a piezoelectric beam) are demonstrated. Thermal effects to sensing and control are discussed.

DEFINITIONS

It is assumed that a generic piezoelectric shell continuum is defined in a curvilinear tri–orthogonal coordinate system in

which the α_1 and α_2 define the neutral surface and α_3 defines the normal. Figure 1. Since the shell is thin the electric field E_3 is considered across the shell thickness and the external electric charge Q_3 is on the top and bottom surfaces only. In this section, assumptions and constitutive equations are defined. (Note that this shell is generic, which can be simplified to a broad class of shell and non—shell geometries. Examples are demonstrated in case studies.)
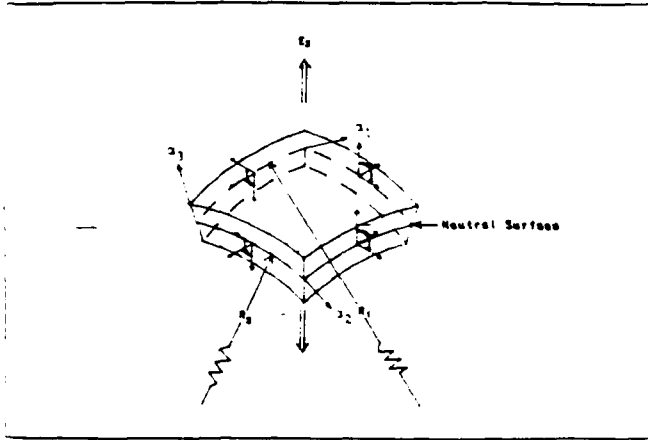

Fig.1 A piezoelectric shell continuum.

The constitutive equation of piezothermoelasticity is defined as

$$\{T\} = [c]\{S\} - [e]^t\{E\} - \{\lambda\}\Delta t_p, \tag{1}$$
$$\{D\} = [e]\{S\} + [\epsilon]\{E\} + \{p\}\Delta t_p, \tag{2}$$

where $\{T\}$ is a stress vector; $[c]$ is the elastic moduli matrix; $[e]$ is the piezoelectric constant; $\{\lambda\} = [s]^{-1}\{\gamma\}$; $[s]$ is the elastic compliance matrix; $\{\gamma\}$ is the coefficient of thermal expansion; $\{D\}$ is the electric displacement vector; $\{S\}$ is the mechanical strain vector; $[\epsilon]$ is the dielectric constant matrix; $\{E\}$ is the electric field vector; $\{p\}$ is the pyroelectric constant; and Δt_p is the temperature change. It is assumed that the piezothermoelastic behaviors are instantly balanced in mechanical, electric, and thermal fields and a quasi—static approximation can be applied. For a piezoelectric shell with a hexagonal symmetrical structure (class $C_{6v} = 6mm$), the elastic moduli $[c]$ matrix is defined by

$$[c_{ij}] = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}, \tag{3}$$

where $c_{11} = (Y/1-\mu^2)$, $c_{12} = (Y\mu/1-\mu^2)$, $c_{66} = \frac{1}{2}(c_{11}-c_{12}) = [Y/2(1+\mu)]$, where μ is Poisson's ratio and Y is Young's modulus. (Note that c_{13}, c_{33}, c_{44} are neglected for thin piezoelectric shells with ineffective in—plane shear constants.) Piezoelectric constant $[e]$ and dielectric constant $[\epsilon]$ matrices are defined by

$$[e_{ij}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e_{15} & e_{15} & 0 \\ e_{13} & e_{13} & e_{33} & 0 & 0 & 0 \end{bmatrix}, \tag{4}$$

$$[\epsilon_{ij}] = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{11} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix} \tag{5}$$

It is assumed that the piezoelectric shell is thin as compared with the other two in—plane dimensions. The transverse shear deformations and rotary inertias are negligible. Thus, the displacement $(U_i, i = 1,2)$ of any given point in the shell continuum can be represented as a summation of the component due to contraction/expansion of the neutral surface and the component due to bending:

$$U_i(\alpha_1,\alpha_2,\alpha_3) = u_i(\alpha_1,\alpha_2) + \alpha_3\beta_i(\alpha_1,\alpha_2),$$
$$i = 1.2.3. \tag{6}$$

where β_i denotes the bending angle and $\beta_3 = 0$. α_3 defines the distance measured from the neutral surface. Based on Kirchhoff—Love assumptions, the transverse shear strains S_{13} and S_{23} are negligible, i.e. $S_{13} = 0$ and $S_{23} = 0$. Thus, the two bending angles can be derived as:

$$\beta_1 = \frac{u_1}{R_1} - \frac{1}{A_1}\frac{\partial u_3}{\partial \alpha_1}, \tag{7}$$

$$\beta_2 = \frac{u_2}{R_2} - \frac{1}{A_2}\frac{\partial u_3}{\partial \alpha_2}. \tag{8}$$

Note that the transverse displacement U_3 is independent of thickness, i.e., $U_3 = u_3(\alpha_1,\alpha_2)$ and the transverse strain S_{33} can thus be neglected, except where a concentrated load is applied. The mechanical strains of the thin shell consist of an in—plane membrane strain component S_{ij}^0 and an out—of—plane bending component k_{ij}:

$$S_{11} = S_{11}^0 + \alpha_3 k_{11}, \tag{9-a}$$
$$S_{22} = S_{22}^0 + \alpha_3 k_{22}, \tag{9-b}$$
$$S_{12} = S_{12}^0 + \alpha_3 k_{12}. \tag{9-c}$$

The membrane and bending strains, S_{ij}^0 and k_{ij}, are defined as follows:

$$S_{11}^0 = \frac{1}{A_1}\frac{\partial u_1}{\partial \alpha_1} + \frac{u_2}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} + \frac{u_3}{R_1}, \tag{10-a}$$

$$S_{22}^0 = \frac{1}{A_2}\frac{\partial u_2}{\partial \alpha_2} + \frac{u_1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1} + \frac{u_3}{R_2}, \tag{10-b}$$

$$S_{12}^0 = \frac{A_2}{A_1}\frac{\partial}{\partial \alpha_1}\left[\frac{u_2}{A_2}\right] + \frac{A_1}{A_2}\frac{\partial}{\partial \alpha_2}\left[\frac{u_1}{A_1}\right], \tag{10-c}$$

$$k_{11} = \frac{1}{A_1}\frac{\partial \beta_1}{\partial \alpha_1} + \frac{\beta_2}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}, \tag{11-a}$$

$$k_{22} = \frac{1}{A_2}\frac{\partial \beta_2}{\partial \alpha_2} + \frac{\beta_1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}, \tag{11-b}$$

$$k_{12} = \frac{A_2}{A_1}\frac{\partial}{\partial \alpha_1}\left[\frac{\beta_2}{A_2}\right] + \frac{A_1}{A_2}\frac{\partial}{\partial \alpha_2}\left[\frac{\beta_1}{A_1}\right], \tag{11-c}$$

where βs are defined in Eqs.(7) and (8). Note that there is no shear strain on the α_1 face such that there is no induced electric field in the α_1 and the α_2 directions. Considering the piezothermoelastic constitutive equations and the stress—strain relations of thin shells, one can define the mechanical stress T_{ij} induced by the mechanical strains, the electric displacement S_i induced by strains, and the stress E_i induced by electric fields.

$$T_{11} = c_{11}S_{11} - c_{12}S_{22}, \tag{12-a}$$
$$T_{22} = c_{12}S_{11} - c_{11}S_{22}, \tag{12-b}$$
$$T_{33} = 0, \tag{12-c}$$
$$T_{12} = c_{66}S_{12}. \tag{12-d}$$

$$\dot{T}_{13} = \dot{T}_{23} = 0 . \qquad (12-e)$$

$$\dot{S}_1 = \dot{S}_2 = 0 , \qquad (13-a)$$

$$\dot{S}_3 = e_{31}S_{11} + e_{31}S_{22} , \qquad (13-b)$$

$$\dot{E}_1 = \dot{E}_2 = e_{31}E_3 , \qquad (14-a)$$

$$\dot{E}_3 = e_{33}E_3 , \qquad (14-b)$$

$$\dot{E}_4 = \dot{E}_5 = \dot{E}_6 = 0 . \qquad (14-c)$$

These terms will be used in conjunctions with the energy expressions and the variational equations.

FORCES AND MOMENTS

In this section, all forces and moments introduced by mechanical, electric, and thermal effects are defined. These force and moment components will be used in Hamilton's equation when deriving the shell piezothermoelastic equations. The mechanical membrane forces are

$$N^m_{11} = \int_{\alpha_3} \dot{T}_{11}d\alpha_3 = K(S^0_{11}+\mu S^0_{22}) , \qquad (15-a)$$

$$N^m_{22} = \int_{\alpha_3} \dot{T}_{22}d\alpha_3 = K(S^0_{22}+\mu S^0_{11}) , \qquad (15-b)$$

$$N^m_{12} = \int_{\alpha_3} \dot{T}_{12}d\alpha_3 = N^m_{21} = \frac{K(1-\mu)}{2}S^0_{12} . \qquad (15-c)$$

where $K = Yh/(1-\mu^2)$ is the membrane stiffness and N^m_{ij} is the total force acting on the ith face in the jth direction due to mechanical effects. The mechanical bending moments are

$$M^m_{11} = \int_{\alpha_3} \dot{T}_{11}\alpha_3d\alpha_3 = D(k_{11}+\mu k_{22}) , \qquad (16-a)$$

$$M^m_{22} = \int_{\alpha_3} \dot{T}_{22}\alpha_3d\alpha_3 = D(k_{22}+\mu k_{11}) , \qquad (16-b)$$

$$M^m_{12} = \int_{\alpha_3} \dot{T}_{12}\alpha_3d\alpha_3 = M^m_{21} = \frac{D(1-\mu)}{2}k_{12} , \qquad (16-c)$$

$$M^m_{13} = M^m_{23} = 0 , \qquad (16-d)$$

where $D = \frac{Yh^3}{12(1-\mu^2)}$ is the bending stiffness and M^m_{ij} is the total bending moment on the ith face in the jth direction due to the mechanical effects. The mechanical transverse shear forces Q^m_{i3} are

$$Q^m_{13} = \int_{\alpha_3} \dot{T}_{13}d\alpha_3 , \qquad (17-a)$$

$$Q^m_{23} = \int_{\alpha_3} \dot{T}_{23}d\alpha_3 . \qquad (17-b)$$

Using Eq.(2), one can derive the electric membrane forces:

$$N^e_{11} = \int_{\alpha_3} e_{31}E_3d\alpha_3$$
$$= -\frac{e_{31}}{\epsilon_{33}}hQ_3 - \frac{e_{31}}{\epsilon_{33}}hp_3\Delta t_p - \frac{e_{31}^2}{\epsilon_{33}}(S^0_{11} - S^0_{22})h , \qquad (18-a)$$

$$N^e_{22} = \int_{\alpha_3} e_{31}E_3d\alpha_3$$
$$= -\frac{e_{31}}{\epsilon_{33}}hQ_3 - \frac{e_{31}}{\epsilon_{33}}hp_3\Delta t_p - \frac{e_{31}^2}{\epsilon_{33}}(S^0_{11}+S^0_{22})h . \qquad (18-b)$$

$$N^e_{12} = 0 . \qquad (18-c)$$

where the first term is contributed by the converse effect, the second term by the pyroelectric effect (temperature), the third term by the elastic strains via the direct effect. The electric bending moments are

$$M^e_{11} = \int_{\alpha_3} e_{31}E_3\alpha_3d\alpha_3$$
$$= -\frac{h^3}{12}\frac{e_{31}^2}{\epsilon_{33}}(k_{11}+k_{22}) . \qquad (19-a)$$

$$M^e_{22} = \int_{\alpha_3} e_{31}E_3\alpha_3d\alpha_3$$
$$= -\frac{h^3}{12}\frac{e_{31}^2}{\epsilon_{33}}(k_{11}+k_{22}) , \qquad (19-b)$$

$$M^e_{12} = M^e_{13} = M^e_{23} = 0 , \qquad (19-c)$$

where M^e_{ij} is the total electric bending moment on the ith face in the jth direction due to the converse piezoelectric effect. The electric transverse shear forces are $Q^e_{13} = Q^e_{23} = 0$. There is no shear forces in the α_3 direction due to the electric effects. The thermal membrane forces N^t_{ii} are defined by

$$N^t_{11} = \int_{\alpha_3} \lambda_1\Delta t_pd\alpha_3 = h\lambda_1\Delta t_p , \qquad (20-a)$$

$$N^t_{22} = \int_{\alpha_3} \lambda_2\Delta t_pd\alpha_3 = h\lambda_2\Delta t_p . \qquad (20-b)$$

The thermal bending moments are

$$M^t_{11} = \int_{\alpha_3} \lambda_1\Delta t_p\alpha_3d\alpha_3 = 0 , \qquad (21-a)$$

$$M^t_{22} = \int_{\alpha_3} \lambda_2\Delta t_p\alpha_3d\alpha_3 = 0 . \qquad (21-b)$$

It is noted that the piezoelectric continua experience only in-plane thermal expansion/contraction and no bending moments in a uniformly distributed thermal field.

HAMILTON'S PRINCIPLE AND PIEZOTHERMOELASTIC EQUATIONS

In this section, piezothermoelastic equations in three principal directions will be derived using Hamilton's principle and the variational procedures. Hamilton's principle gives (Tzou & Zhong, 1990)

$$\delta\int_{t_0}^{t_1}\int_V \left[\frac{1}{2}\rho\dot{U}_j\dot{U}_j - H(S_{kj},E_j)\right]dVdt$$
$$+ \int_{t_0}^{t_1}\int_S (\bar{t}_j\delta U_j - \overline{Q}\delta\varphi)dSdt = 0 , \qquad (22)$$

where H is the electric enthalpy; ρ is the mass density; U_j is the displacement; E_j is the electric field; $\bar{t}_j$ is the surface traction in the α_j direction; S_{kj} is the strain on the kth face and in the jth direction; $\overline{Q}$ is the surface charge; and φ is the electric potential. The electric fields in the curvilinear coordinate system are 1) $E_1 = -\frac{1}{A_1(1+\alpha_3/R_1)}\frac{\partial\varphi}{\partial\alpha_1}$, 2) $E_2 = -\frac{1}{A_2(1+\alpha_3/R_2)}\frac{\partial\varphi}{\partial\alpha_2}$, and 3) $E_3 = -\frac{\partial\varphi}{\partial\alpha_3}$, where R_1 and R_2 are the radii of curvatures, and A_1 and A_2 are Lame's parameters. These define the electric fields as the gradient of the electric potential. The electric enthalpy is

defined as $H = \frac{1}{2}[\{S\}^t\{T\} - \{E\}^t\{D\}]$ (Tzou & Zhong, 1990). Using the piezothermoelastic constitutive equations, one can derive

$$H = \frac{1}{2}\{S\}^t[c]\{S\} - \{E\}^t[e]\{S\} - \frac{1}{2}\{E\}^t[\epsilon]\{E\}$$
$$- \{S\}^t\{\lambda\}\Delta t_p - \{E\}^t\{p\}\Delta t_p . \tag{23}$$

Substituting the strain–stress expressions into the electric enthalpy gives

$$H = \frac{1}{2}(T_{11}S_{11} + T_{22}S_{22} + T_{12}S_{12}) - e_{31}(S_{11} + S_{22})E_3$$
$$- \frac{1}{2}(\epsilon_{33}E_3{}^2) - (\lambda_1 S_{11} + \lambda_2 S_{22} + \lambda_3 S_{33} + p_3 E_3)\Delta t_p . \tag{24}$$

Substituting the electric enthalpy and all other energy expressions into Hamilton's equation and collecting all like terms in the variational equation, one can derive the piezothermoelastic and vibration equations in three principal directions.

$$\frac{\partial[(N_{11}^m - N_{11}^e - N_{11}^t)A_2]}{\partial \alpha_1} + \frac{\partial[N_{12}^m A_1]}{\partial \alpha_2} - (N_{22}^m - N_{22}^e - N_{22}^t)\frac{\partial A_2}{\partial \alpha_1}$$
$$+ Q_{13}^m\frac{A_1 A_2}{R_1} + N_{12}^m\frac{\partial A_1}{\partial \alpha_2} = \rho h A_1 A_2\frac{\partial^2 u_1}{\partial t^2} . \tag{25-a}$$

$$\frac{\partial[N_{12}^m A_2]}{\partial \alpha_1} + \frac{\partial[(N_{22}^m - N_{22}^e - N_{22}^t)A_1]}{\partial \alpha_2} - (N_{11}^m - N_{11}^e - N_{11}^t)\frac{\partial A_1}{\partial \alpha_2}$$
$$+ Q_{23}^m\frac{A_1 A_2}{R_2} + N_{12}^m\frac{\partial A_2}{\partial \alpha_1} = \rho h A_1 A_2\frac{\partial^2 u_2}{\partial t^2} . \tag{26-a}$$

$$\frac{\partial[Q_{13}^m A_2]}{\partial \alpha_1} + \frac{\partial[Q_{23}^m A_1]}{\partial \alpha_2} - (N_{11}^m - N_{11}^e - N_{11}^t)\frac{A_1 A_2}{R_1}$$
$$- (N_{22}^m - N_{22}^e - N_{22}^t)\frac{A_1 A_2}{R_2} = \rho h A_1 A_2\frac{\partial^2 u_3}{\partial t^2} . \tag{27-a}$$

where h is the thickness of piezoelectric shell. The superscripts m, e, and t respectively denote mechanical, electric, and thermal components. Q_{13}^m and Q_{23}^m in Eqs.(25)–(27) are defined by

$$Q_{13}^m A_1 A_2 = \frac{\partial[(M_{11}^m - M_{11}^e)A_2]}{\partial \alpha_1} + \frac{\partial[M_{12}^m A_1]}{\partial \alpha_2}$$
$$- (M_{22}^m - M_{22}^e)\frac{\partial A_2}{\partial \alpha_1} + M_{12}^m\frac{\partial A_1}{\partial \alpha_2} , \tag{28}$$

$$Q_{23}^m A_1 A_2 = \frac{\partial[M_{12}^m A_2]}{\partial \alpha_1} + \frac{\partial[(M_{22}^m - M_{22}^e)A_1]}{\partial \alpha_2}$$
$$- (M_{11}^m - M_{11}^e)\frac{\partial A_1}{\partial \alpha_2} + M_{12}^m\frac{\partial A_2}{\partial \alpha_1} . \tag{29}$$

Note that the the equation of motions include the mechanical forces/moments (N_{ij}^m/M_{ij}^m), electric forces/moments (N_{ij}^e/M_{ij}^e), and thermal induced forces (N_{ii}^t). As discussed previously, thermal expansions/contractions in three principal directions are considered. It is observed that the uniform temperature variation does not contribute any thermal moments, which will not be the case in non–uniform temperature variations. These system equations can be solved, with the appropriate boundary conditions and external excitations (mechanical, electric, and/or thermal), to describe the exact piezothermoelastic behaviors of the piezoelectric shell. In structural control applications, the electric related components can be used as control forces and moments to alter system characteristics (Tzou & Zhong, 1991b). Rearranging Eqs.(25–27) and moving all electric related terms (control terms) to the right (Tzou, 1991), one can derive

$$\frac{\partial[(N_{11}^m - N_{11}^t)A_2]}{\partial \alpha_1} + \frac{\partial[N_{12}^m A_1]}{\partial \alpha_2} - (N_{22}^m - N_{22}^t)\frac{\partial A_2}{\partial \alpha_1}$$
$$+ \frac{1}{R_1}\left[\frac{\partial[(M_{11}^m)A_2]}{\partial \alpha_1} + \frac{\partial[M_{12}^m A_1]}{\partial \alpha_2} - (M_{22}^m)\frac{\partial A_2}{\partial \alpha_1}\right]$$
$$+ M_{12}^m\frac{\partial A_1}{\partial \alpha_2} + N_{12}^m\frac{\partial A_1}{\partial \alpha_2} - \rho h A_1 A_2\frac{\partial^2 u_1}{\partial t^2}$$
$$= \frac{\partial(N_{11}^e A_2)}{\partial \alpha_1} - N_{22}^e\frac{\partial A_2}{\partial \alpha_1} + \frac{1}{R_1}\left[\frac{\partial(M_{11}^e)A_2}{\partial \alpha_1}\right.$$
$$- (M_{22}^e)\frac{\partial A_2}{\partial \alpha_1}\Big] , \tag{25-b}$$

$$\frac{\partial[N_{12}^m A_2]}{\partial \alpha_1} + \frac{\partial[(N_{22}^m - N_{22}^t)A_1]}{\partial \alpha_2} - (N_{11}^m - N_{11}^t)\frac{\partial A_1}{\partial \alpha_2}$$
$$+ \frac{1}{R_2}\left[\frac{\partial[M_{12}^m A_2]}{\partial \alpha_1} + \frac{\partial[(M_{22}^m)A_1]}{\partial \alpha_2} - (M_{11}^m)\frac{\partial A_1}{\partial \alpha_2}\right]$$
$$+ M_{12}^m\frac{\partial A_2}{\partial \alpha_1} + N_{12}^m\frac{\partial A_2}{\partial \alpha_1} - \rho h A_1 A_2\frac{\partial^2 u_2}{\partial t^2}$$
$$= \frac{\partial(N_{22}^e)A_1}{\partial \alpha_2} - N_{11}^e\frac{\partial A_1}{\partial \alpha_2} + \frac{1}{R_2}\left[\frac{\partial(M_{22}^e)A_1}{\partial \alpha_2}\right.$$
$$- (M_{11}^e)\frac{\partial A_1}{\partial \alpha_2}\Big] , \tag{26-b}$$

$$\frac{1}{A_1 R_1}\left[\frac{\partial^2[(M_{11}^m)A_2]}{\partial \alpha_1{}^2} + \frac{\partial^2[M_{12}^m A_1]}{\partial \alpha_1 \partial \alpha_2} - (M_{22}^m)\frac{\partial^2 A_2}{\partial \alpha_1{}^2}\right.$$
$$+ M_{12}^m\frac{\partial^2 A_1}{\partial \alpha_1 \partial \alpha_2}\Big] + \frac{1}{A_2 R_2}\left[\frac{\partial^2[M_{12}^m A_2]}{\partial \alpha_1 \partial \alpha_2} + \frac{\partial^2[(M_{22}^m)A_1]}{\partial \alpha_2{}^2}\right.$$
$$- (M_{11}^m)\frac{\partial^2 A_1}{\partial \alpha_1 \partial \alpha_2} + M_{12}^m\frac{\partial^2 A_2}{\partial \alpha_1 \partial \alpha_2}\Big] - (N_{11}^m - N_{11}^t)\frac{A_1 A_2}{R_1}$$
$$- (N_{22}^m - N_{22}^t)\frac{A_1 A_2}{R_2} - \rho h A_1 A_2\frac{\partial^2 u_3}{\partial t^2}$$
$$= - N_{11}^e\frac{A_1 A_2}{R_1} - N_{22}^e\frac{A_1 A_2}{R_2} + \frac{1}{A_1 R_1}\left[\frac{\partial^2(M_{11}^e)A_2}{\partial \alpha_1{}^2}\right.$$
$$- (M_{22}^e)\frac{\partial^2 A_2}{\partial \alpha_1{}^2}\Big] + \frac{1}{A_2 R_2}\left[\frac{\partial^2(M_{22}^e)A_1}{\partial \alpha_2{}^2} - (M_{11}^e)\frac{\partial^2 A_1}{\partial \alpha_2{}^2}\right] . \tag{27-b}$$

Note that electric related terms can be used as control terms to actively change the shell dynamics. In addition, all terms with a constant either $1/R_1$ or $1/R_2$ vanish if the radius of curvature is infinite, e.g., flat plates (Tzou, 1991). The charge equation of electrostatics of the piezoelectric shell is derived:

$$\frac{\partial[(e_{31}S_{11} + e_{31}S_{22} + \epsilon_{33}E_3 + p_3\Delta t_p)A_1 A_2]}{\partial \alpha_3} = 0 , \tag{30}$$

which implies that the quantity $[e_{31}(S_{11} + S_{22}) + \epsilon_{33}E_3 + p_3\Delta t_p]$ $A_1 A_2$ is equal to a constant and the thickness variation is equal to zero. Note that this equation can be used to estimate an electric output as functions of induced mechanical strains and temperature variation, i.e., $E_3 = -(1/\epsilon_{33})[e_{31}(S_{11} + S_{22}) + p_3\Delta t_p]$, in an open–circuit condition (Tzou, Zhong, 1991a).

BOUNDARY CONDITIONS

Boundary conditions are directly derived from the variational equation (Appendix). The boundary conditions are defined by the surface traction forces and the surface charge. (Note that other types of boundary forces and moments, such as spring supported boundaries, fixed/hinged boundaries, etc., can also be accommodated.)

Mechanical Boundary Conditions

Mechanical boundary conditions defined by either force/moment or displacement/rotation are summarized in Table 1 in which terms with a superscript * denote external boundary components.

Table 1. Mechanical boundary conditions.

Force B.C.'s	Disp B.C.
$N^m_{kk} - N^e_{kk} - N^t_{kk} = N^*_{kk}$	$u_k = u^*_k$
$M^m_{kk} - M^e_{kk} = M^*_{kk}$	$\beta_k = \beta^*_k$
$V_{k3} = V^*_{k3}$	$u_3 = u^*_3$
$Q_{kt} = Q^*_{kt}$	$u_t = u^*_t$

where $k = 1.2$, the subscript t denotes the tangential direction (i.e., $t = 2$ if $k = 1$ and vice versa). It is observed that the thermal induced membrane force only occurs in the principal direction. The mechanical shear stress resultants are defined as

$$V_{13} = Q^m_{13} - \frac{1}{A_2} \frac{\partial M^m_{12}}{\partial \alpha_2} \text{ and}$$

$$V_{23} = Q^m_{23} + \frac{1}{A_1} \frac{\partial M^m_{21}}{\partial \alpha_1}. \qquad (31-a,b)$$

$$Q_{12} = N^m_{12} + \frac{M^m_{12}}{R_2} \text{ and}$$

$$Q_{21} = N^m_{21} + \frac{M^m_{21}}{R_1}. \qquad (32-a,b)$$

Again, there is no electrically induced shear components because the in-plane twisting effect is neglected. Note that usually only either force boundary conditions or displacement boundary conditions are selected for a given physical boundary condition. For a totally fixed edge at $\alpha_1 = \alpha_1^*$ (i.e., no motion allowed), the boundary conditions are: $u_1 = 0$, $\beta_1 = 0$, $u_3 = 0$, and $u_2 = 0$. For a totally free edge at $\alpha_2 = \alpha_2^*$, i.e., no external forces and moments, the boundary conditions at $\alpha_2 = \alpha_2^*$ are: $N^m_{22} - N^e_{22} = 0$, $M^m_{22} - M^e_{22} = 0$, $V_{23} = 0$, and $T_{21} = 0$. In the case where the surface traction forces t_{ij} are defined, the boundary membrane forces are

$$N^*_{11} = \int_{\alpha_3} t_{11} d\alpha_3, \qquad (33-a)$$

$$N^*_{22} = \int_{\alpha_3} t_{22} d\alpha_3, \qquad (33-b)$$

$$N^*_{12} = \int_{\alpha_3} t_{12} d\alpha_3, \qquad (33-c)$$

$$N^*_{21} = \int_{\alpha_3} t_{21} d\alpha_3, \qquad (33-d)$$

where N^*_{ij} is the total force on the ith face in the jth direction due to the surface tractions. The induced boundary bending moments M^*_{ij} are

$$M^*_{11} = \int_{\alpha_3} t_{11}\alpha_3 d\alpha_3, \qquad (34-a)$$

$$M^*_{22} = \int_{\alpha_3} t_{22}\alpha_3 d\alpha_3. \qquad (34-b)$$

$$M^*_{12} = \int_{\alpha_3} t_{12}\alpha_3 d\alpha_3, \qquad (34-c)$$

$$M^*_{21} = \int_{\alpha_3} t_{21}\alpha_3 d\alpha_3. \qquad (34-d)$$

Accordingly, the boundary transverse shear forces Q^*_{ij} are

$$Q^*_{13} = Q^*_{31} = \int_{\alpha_3} t_{13} d\alpha_3. \qquad (35-a)$$

$$Q^*_{23} = Q^*_{32} = \int_{\alpha_3} t_{23} d\alpha_3 \qquad (35-b)$$

where Q^*_{ij} is the shear force on the ith face in the jth direction.

Electric Boundary Condition

The electric boundary condition is defined as

$$e_{31}S_{11} + e_{31}S_{22} + \epsilon_{33}E_3 + p_3\Delta t_p + Q^*_3 = 0 \qquad (36)$$

It is observed that the total surface charge including the mechanical, electric, and temperature effects is equal to the external surface charge Q^*_3.

Note that the piezothermoelastic equations for the thin shell continuum and the boundary conditions can be reduced to conventional elastic shell equations by neglecting all electric and thermal coupling terms (Soedel, 1981). Again, transverse shear deformation and rotatory inertia effects were not considered.

PIEZOTHERMOELECTRICITY OF SIMPLIFIED GEOMETRIES

The piezothermoelastic theory derived above is for a generic piezoelectric shell continuum exposed to mechanical, thermal, and electric fields. The generic shell was defined in a curvilinear tri-orthogonal coordinate system defined by α_1, α_2, and α_3 axes. The in-plane two axes define the neutral surface experiencing only membrane effects. Each of the in-plane axis is defined by its radius of curvature, e.g., R_1 for α_1 and R_2 for α_2. In addition, there are two Lame's parameters (A_1 and A_2) defined by a *fundamental form*: $(ds)^2 = A_1^2 (d\alpha_1)^2 + A_2^2 (d\alpha_2)^2$. For a given geometry, R_1 and R_2 can usually be directly observed from the coordinate system and A_1 and A_2 can be derived from the fundamental form. Substituting the four parameters into the generic shell equation and simplifying them accordingly, one can derive the corresponding piezothermoelastic equations and boundary conditions for the geometry. In this section, these procedures are used to derive the piezothermoelastic equations for 1) a piezoelectric cylindrical shell and 2) a piezoelectric beam.

Example-1: Piezoelectric Cylindrical Shell

It is assumed that the cylindrical shell is defined in a cylindrical coordinate system in which x axis (α_1) is aligned with the height and its radius of curvature $R_1 = \infty$. The second axis θ (α_2) defines the circumferential direction which has a radius of curvature $R_2 = R$. Note that the x and θ axes constitute the neutral surface. The third axis α_3 is normal to the neutral surface. Figure 2 illustrates the piezoelectric cylinder and its coordinate system. Piezothermoelastic effects of the cylindrical shell will be discussed.
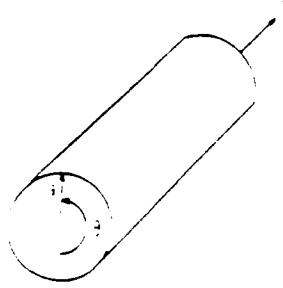
Fig.2 A piezoelectric cylindrical shell.

The fundamental form of the cylinder is

$$(ds)^2 = (1)^2(dx)^2 + R^2(d\theta)^2 . \tag{37}$$

Thus, $A_1 = 1$, $A_2 = R$, $R_1 = \infty$, $R_2 = R$, $\alpha_1 = x$, $\alpha_2 = \theta$. Substituting these parameters into the member/bending strain expressions in Section-2, one can derive the membrane S_{ij}^0 and bending strains k_{ij} for the cylindrical shell. Thus, the total strains are

$$S_{11} = \frac{\partial u_x}{\partial x} - \alpha_3 \frac{\partial^2 u_3}{\partial x^2} . \tag{38-a}$$

$$S_{22} = \frac{1}{R}\left[\frac{\partial u_\theta}{\partial \theta} + u_3\right] + \frac{\alpha_3}{R^2}\left[\frac{\partial u_\theta}{\partial \theta} - \frac{\partial^2 u_3}{\partial \theta^2}\right] . \tag{38-b}$$

$$S_{12} = \left[\frac{\partial u_\theta}{\partial x} + \frac{1}{R}\frac{\partial u_x}{\partial \theta}\right] + \frac{\alpha_3}{R}\left[\frac{\partial u_\theta}{\partial x} - 2\frac{\partial^2 u_3}{\partial \theta \partial x}\right] \tag{38-c}$$

Substituting the strains into the stress equations and consequently into the force/moment equations, one can derive the force/moment for the cylindrical shell:

$$N_{11}^m = K\left[\frac{\partial u_x}{\partial x} + \frac{\mu}{R}\left[\frac{\partial u_\theta}{\partial \theta} + u_3\right]\right] . \tag{39-a}$$

$$N_{22}^m = K\left[\frac{1}{R}\left[\frac{\partial u_\theta}{\partial \theta} + u_3\right] + \mu\frac{\partial u_x}{\partial x}\right] . \tag{39-b}$$

$$N_{12}^m = \frac{K(1-\mu)}{2}\left[\frac{\partial u_\theta}{\partial x} + \frac{1}{R}\frac{\partial u_x}{\partial \theta}\right] \tag{39-c}$$

$$M_{11}^m = D\left[-\frac{\partial^2 u_3}{\partial x^2} + \frac{\mu}{R^2}\left[\frac{\partial u_\theta}{\partial \theta} - \frac{\partial^2 u_3}{\partial \theta^2}\right]\right] . \tag{40-a}$$

$$M_{22}^m = D\left[\frac{1}{R^2}\left[\frac{\partial u_\theta}{\partial \theta} - \frac{\partial^2 u_3}{\partial \theta^2}\right] - \mu\frac{\partial^2 u_3}{\partial x^2}\right] . \tag{40-b}$$

$$M_{12}^m = \frac{D(1-\mu)}{2}\left[\frac{1}{R}\left[\frac{\partial u_\theta}{\partial x} - 2\frac{\partial^2 u_3}{\partial \theta \partial x}\right]\right] \tag{40-c}$$

$$N_{11}^e = -\frac{e_{31}^2}{\epsilon_{33}}h\left[\frac{\partial u_x}{\partial x} + \frac{1}{R}\left[\frac{\partial u_\theta}{\partial \theta} + u_3\right]\right]$$
$$-\frac{e_{31}}{\epsilon_{33}}h\left[p_3 \Delta t_p + Q_3\right] . \tag{41-a}$$

$$N_{22}^e = N_{11}^e \tag{41-b}$$

$$M_{11}^e = -\frac{e_{31}^2}{\epsilon_{33}}\frac{h^3}{12}\left[-\frac{\partial^2 u_3}{\partial x^2} + \frac{1}{R^2}\left[\frac{\partial u_\theta}{\partial \theta} - \frac{\partial^2 u_3}{\partial \theta^2}\right]\right] . \tag{42-a}$$

$$M_{22}^e = M_{11}^e \tag{42-b}$$

$$N_{11}^t = \lambda_1 \Delta t_p . \tag{43-a}$$

$$N_{22}^t = \lambda_2 \Delta t_p \tag{43-b}$$

Note that the superscripts m, e, and t are for the mechanical, electric, and thermal effects respectively. Substituting the force and moment terms into the Q_{13}^m and Q_{23}^m equations, one can derive

$$Q_{13}^m = \left[-D - \frac{e_{31}^2}{\epsilon_{33}}\frac{h^3}{12}\right]\frac{\partial^3 u_3}{\partial x^3} - \left[\frac{D(1+\mu)}{2R^2} - \frac{e_{31}^2 h^3}{12R^2}\right]$$
$$\cdot\frac{\partial^2 u_\theta}{\partial \theta \partial x} + \left[-\frac{D}{R^2} - \frac{e_{31}^2}{\epsilon_{33}}\frac{h^3}{12R^2}\right]\frac{\partial^3 u_3}{\partial x \partial \theta^2} . \tag{44}$$

$$Q_{23}^m = \left[-\frac{D}{R^3} - \frac{e_{31}^2}{\epsilon_{33}}\frac{h^3}{12R^3}\right]\frac{\partial^3 u_3}{\partial \theta^3} + \left[\frac{D(1-\mu)}{2R}\right]\frac{\partial^2 u_\theta}{\partial x^2}$$
$$+ \left[-\frac{D}{R} - \frac{e_{31}^2 h^3}{\epsilon_{33} 12R}\right]\frac{\partial^3 u_3}{\partial x^2 \partial \theta} + \left[\frac{D}{R^3} + \frac{e_{31}^2}{\epsilon_{33}}\frac{h^3}{12R^3}\right]\frac{\partial^2 u_\theta}{\partial \theta^2} \tag{45}$$

Thus, the piezothermoelastic equations in three principal directions for the piezoelectric cylindrical shell are derived.

$$\frac{\partial[(N_{11}^m - N_{11}^e - N_{11}^t)R]}{\partial x} + \frac{\partial[N_{12}^m]}{\partial \theta} = \rho h R\frac{\partial^2 u_x}{\partial t^2} . \tag{46}$$

$$\frac{\partial[N_{12}^m R]}{\partial x} + \frac{\partial[N_{22}^m - N_{22}^e - N_{22}^t]}{\partial \theta} + Q_{23}^m = \rho h R\frac{\partial^2 u_\theta}{\partial t^2} , \tag{47}$$

$$\frac{\partial[Q_{13}^m R]}{\partial x} + \frac{\partial[Q_{23}^m]}{\partial \theta} - [N_{22}^m - N_{22}^e - N_{22}^t] = \rho h R\frac{\partial^2 u_3}{\partial t^2} . \tag{48}$$

It is observed that the thermal effects only contribute to the membrane forces. Removing the electric and thermal related terms, one can simplify the system equations to those corresponding to an elastic cylindrical shell.

Using the charge boundary condition, one can define the electric field strength E_3 at the location α_3 above/below the neutral surface as a function of the mechanical strains, temperature effect, and charge effect.

$$E_3 = -\frac{e_{31}}{\epsilon_{33}}\left[\frac{\partial u_x}{\partial x} + \frac{1}{R}\left[\frac{\partial u_\theta}{\partial \theta} + u_3\right] - \alpha_3\frac{\partial^2 u_3}{\partial x^2}\right.$$
$$\left. + \frac{\alpha_3}{R^2}\left[\frac{\partial u_\theta}{\partial \theta} - \frac{\partial^2 u_3}{\partial \theta^2}\right]\right] - \frac{p_3 \Delta t_p}{\epsilon_{33}} - \frac{Q_3}{\epsilon_{33}} \tag{49}$$

The electric field strength is contributed by the direct piezoelectric effect (the first term), the pyroelectric effect (the second term), and the external surface charge (the third term) as defined in the constitutive equation. Note that the resulting voltage is $V_3 = \int_{\alpha_3} E_3 d\alpha_3$ in an open-circuit condition. The bending components, with α_3 terms, vanish after the integration. It is also observed that the output signal has a temperature related term induced by the pyroelectric effect in sensor applications. Note that it is assumed that the external charge is zero in sensor applications (Tzou & Zhong, 1991a).

Example-2: Piezoelectric Beam

A beam is a special case of an open ring with zero curvature, $R = \infty$. In this case, the α_1 axis is aligned with the longitudinal direction of the cantilever beam, i.e., $\alpha_1 = x$. The second axis is in the width direction, $\alpha_2 = y$. Figure 3 shows the piezoelectric beam. It is assumed that the beam only experiences transverse oscillations, $\alpha_3 = z$. Governing equation and piezothermoelastic behaviors of the beam are discussed.

The fundamental form of the beam is

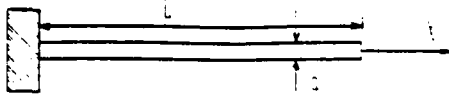$$(ds)^2 = (1)^2(dx)^2 + (1)^2(dy)^2 . \tag{50}$$

$$- \mathbf{YT} - [\frac{e_{31}^2}{\epsilon_{33}}] \frac{\partial^4 u_3}{\partial x^4} = \rho hb \frac{\partial^2 u_3}{\partial t^2} \qquad (57)$$

where $I = \frac{bh^3}{12}$. Note that the elasticity part has one more term contributed by the piezoelectricity. The piezoelectricity contributed elasticity is very small, about 1% for piezoelectric polyvinylidene fluoride polymer (Tzou & Zhong, 1991a). However, the temperature has no contribution to the transverse oscillation because the thermal forces are primarily on the neutral surface, neutral axis in this case. This pyroelectric effect will contribute to the longitudinal oscillation.

The electric field strength at the location α_3 above/below the neutral axis is defined by the external charge, temperature induced pyroelectric effect, and bending strain.

$$E_3 = - \frac{Q_3}{\epsilon_{33}} - \frac{1}{\epsilon_{33}} p_3 \Delta t_p + \left[\frac{e_{31}}{\epsilon_{33}} \alpha_3 \frac{\partial^2 u_3}{\partial x^2} \right] \qquad (58)$$

However, the resultant open–circuit voltage V_3 is, in fact, only contributed by the pyroelectric effect $\{Q_3 = 0$ and $\int_{-h/2}^{h/2} \frac{e_{31}}{\epsilon_{33}} \alpha_3 \frac{\partial^2 u_3}{\partial x^2} \} d\alpha_3 = 0.\}$

SUMMARY AND CONCLUSIONS

A linear piezothermoelastic theory of piezoelectric shell continua was proposed and piezothermoelastic phenomena were evaluated. It was assumed that the electric, thermal, and elastic fields are instantaneously balanced and a quasi–static condition is used in the piezothermoelastic constitutive equations. A generic theory for a piezoelectric thin shell continuum was derived using Hamilton's principle and Kirchhoff Love assumptions. The governing equations show close coupling effects among electric, thermal, and elastic fields. Both mechanical and electric effects contribute to the resultant forces/moments for the shell continuum. However, it was observed that the thermal effect only contributes to the membrane force resultants, not the bending resultants due to a uniform temperature assumption. Thermal induced bending could appear if there is a non–uniform temperature distribution. Note that the electric force/moment resultants in the piezothermoelastic equations can be used to control the shell continuum.

The derived piezothermoelastic equations are generic, which can be simplified to a variety of piezoelectric continua if two radii of curvatures and two Lame's parameters are defined. This simplification was demonstrated in three examples: 1) a cylindrical shell and 2) a beam. Detailed piezothermoelastic phenomena of each geometry were discussed along with the derived governing equations. The same procedure can be applied to a variety of other piezoelectric continua and so as the piezothermoelasticity evaluated. Note that the theory was derived based on linear assumptions and the material nonlinearity was not considered. However, these material constants (e.g., piezoelectric constants, elastic constants, etc.) could vary when temperature variation is significant. Thus, extending the present theory to encompass the material nonlinearity would further enhance the theoretical development and understand more about the complicated behaviors of piezoelectric sensors/actuators operating in non–ideal environments.

Fig.3 A piezoelectric cantilever beam.

where dx and dy are infinitesimal distances in the x and y directions respectively. Thus, $A_1 = 1$, $A_2 = 1$, $R_1 = x$, $R_2 = x$. Since only the bending oscillation is considered, the membrane strains are zeros, i.e., $S_{11}^0 = 0$, $S_{22}^0 = 0$, and $S_{12}^0 = 0$. The bending strain at the α_3 location is defined by $k_{11} = - \frac{\partial^2 u_3}{\partial x^2}$ and $k_{22} = 0$, $k_{12} = 0$. The total strains at α_3 location are defined as

$$S_{11} = - \alpha_3 \frac{\partial^2 u_3}{\partial x^2}, \quad S_{22} = 0, \quad S_{12} = 0 \qquad (51)$$

Again, the beam experiences only transverse oscillation. The membrane (longitudinal) force components are all zeros, i.e., $N_{11}^m = 0$, $N_{22}^m = 0$, $N_{12}^m = 0$. The resultant moments are

$$M_{11}^m = D(k_{11} + \mu k_{22}) = - D \frac{\partial^2 u_3}{\partial x^2} \qquad (52-a)$$

$$M_{22}^m = D(k_{22} + \mu k_{11}) = - \mu D \frac{\partial^2 u_3}{\partial x^2} \qquad (52-b)$$

$$M_{12}^m = \frac{D(1-\mu)}{2} k_{12} = 0 \qquad (52-c)$$

Note that the moment M_{22}^m is primarily introduced by Poisson's effect. The electric force and moment resultants due to the external charge and temperature are

$$N_{11}^e = - h \frac{e_{31}}{\epsilon_{33}} Q_3 - h \frac{e_{31}}{\epsilon_{33}} p_3 \Delta t_p \qquad (53-a)$$

$$N_{22}^e = - h \frac{e_{31}}{\epsilon_{33}} Q_3 - h \frac{e_{31}}{\epsilon_{33}} p_3 \Delta t_p \qquad (53-b)$$

$$M_{11}^e = \frac{h^3}{12} \frac{e_{31}^2}{\epsilon_{33}} \frac{\partial^2 u_3}{\partial x^2} \qquad (54-a)$$

$$M_{22}^e = \frac{h^3}{12} \frac{e_{31}^2}{\epsilon_{33}} \frac{\partial^2 u_3}{\partial x^2} \qquad (54-b)$$

$$N_{11}^t = h \lambda_1 \Delta t_p \qquad (55-a)$$

$$N_{22}^t = h \lambda_1 \Delta t_p \qquad (55-b)$$

Substituting the system parameters and force/moment resultants into the original shell equation, one can derive the transverse piezothermoelastic equation

$$- \left[D + \frac{h^3}{12} \frac{e_{31}^2}{\epsilon_{33}} \right] \frac{\partial^4 u_3}{\partial x^4} = \rho h \frac{\partial^2 u_3}{\partial t^2} \qquad (56)$$

For a beam with a rectangular cross–section (width b and thickness h), the transverse equation of motion is

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